Range Trees

N-Dimensional Dictionary: keys are d-dimensional vectors $(x_0, x_1, \ldots, x_{d-1})$

Range Search query: Give me all entries with

\[ a \leq x_k \leq b, \]
\[ c \leq x_l \leq d \]
\[ e \leq x_m \leq f \]

Finding region of n-dimensional space
One-Dimensional Range Searching

Algorithm treeRangeSearch(k₁, k₂, v)

Input: keys k₁, k₂ and v a node of a BST T
Output: elements in subtree of T with keys between k₁, k₂ inclusive.

if v is an external node then
    return the empty set

if k₁ ≤ key(v) ≤ k₂ then
    \( E_L \leftarrow \text{treeRangeSearch}(k_1, k_2, T.\text{leftChild}(v)) \)
    \( E_R \leftarrow \text{treeRangeSearch}(k_1, k_2, T.\text{rightChild}(v)) \)
    return \( E_L \cup \{\text{element}(v)\} \cup E_R \)

else if key(v) < k₁ then
    return \( \text{treeRangeSearch}(k_1, k_2, T.\text{rightChild}(v)) \)

else if k₂ < key(v) then
    return \( \text{treeRangeSearch}(k_1, k_2, T.\text{leftChild}(v)) \)

\((h, s)\) where h is height, s number of elements returned
Two-dimensional Range trees

eys: \((x_1, x)\)

Gist: Nest 1-dimensional range trees or \(x\) in the nodes of 1-dimensional range trees or \(x_1\)
Quadtrees and Octrees

Two and three dimensional data

[Diagram of a quadtree with nodes A, B, C, D, E]
K-d Trees

Split regions based on distribution of points

Subdivision

Tree structure

The range

Nodes visited in search
Home home on the range...