DIGRAPHS

- Reachability
- Connectivity
- Transitive Closure
- Floyd-Warshall Algorithm
A typical student day

1. wake up
2. cs16 meditation
3. eat
4. work
5. more cs16
6. battletris
7. play
8. cs16 program
9. make cookies for cs16 HTA
10. sleep
11. dream of cs16
What’s a Digraph?

a) A small burrowing animal with long sharp teeth and a unquenchable lust for the blood of computer science majors

b) A distressed graph

c) A directed graph

Each edge goes in one direction

Edge \((a,b)\) goes from \(a\) to \(b\), but not \(b\) to \(a\)

You’re saying, “Yo, how about an example of how we might be enlightened by the use of digraphs!!” – Well, if you insist. . .
Applications

Maps: digraphs handle one-way streets
(especially helpful in Providence)

Thomas J. Watson Jr. Center for Information Technology

ME!
Another Application

*Scheduling:* edge \((a,b)\) means task \(a\) must be completed before \(b\) can be started

Old programmers never die - they just fall into black holes
DAG’s

dag: (noun) dÂ-g

1. Di-Acyl-Glycerol – My favorite snack!
2. “man’s best friend”
   person’s
3. directed acyclic graph

Say What?!

directed graph with no directed cycles

![Diagram of DAG and not a DAG]
Depth-First Search

Same algorithm as for undirected graphs

On a connected digraph, may yield unconnected DFS trees (i.e., a DFS forest)
Reachability

DFS tree rooted at $v$: vertices reachable from $v$ via directed paths
Strongly Connected Digraphs

Each vertex can reach all other vertices
Strongly Connected Components

\{ a, c, g \} \\
\{ f, d, e, b \}
Transitive Closure

Digraph $G^*$ is obtained from $G$ using the rule:

If there is a directed path in $G$ from $a$ to $b$, then add the edge $(a,b)$ to $G^*$.
Computing the Transitive Closure

We can perform DFS starting at each vertex
Time: $O(n(n+m))$

Alternatively ... Floyd-Warshall Algorithm:

If there’s a way to get from $a$ to $b$, and from $b$ to $c$, then there’s a way to get from $a$ to $c$
Example
Floyd-Warshall Algorithm

- this algorithm assumes that methods `areAdjacent` and `insertDirectedEdge` take $O(1)$ time (e.g., adjacency matrix structure)

**Algorithm FloydWarshall(G)**

```plaintext
let v₁ ... vₙ be an arbitrary ordering of the vertices
G₀ = G
for k = 1 to n do
    // consider all possible routing vertices vᵦ
    Gᵦ = Gᵦ₋₁ // these are the only ones you need to store
    for each (i, j = 1, ..., n) (i ≠ j) (i, j ≠ k) do
        // for each pair of vertices vᵢ and vⱼ
        if Gᵦ₋₁.areAdjacent(vᵢ, vᵦ) and Gᵦ₋₁.areAdjacent(vᵦ, vⱼ) then
            Gᵦ.insertDirectedEdge(vᵢ, vⱼ, null)
    return Gₙ
```

- digraph $Gₖ$ is the subdigraph of the transitive closure of $G$ induced by paths with intermediate vertices in the set \{ v₁, ..., vₖ \}

- running time: $O(n^3)$
Example

• digraph G
Example

- digraph $G^*$
Topological Sorting

For each edge \((u,v)\), vertex \(u\) is visited before vertex \(v\)

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Topological Sorting

Topological sorting may not be unique

You make the call!

A B C D
or
A C B D
Topological Sorting

Labels are increasing along a directed path

A digraph has a topological sorting if and only if it is acyclic (i.e., a dag)
Algorithm for Topological Sorting

method TopologicalSort
  if there are more vertices
    let v be a source;
    // a vertex w/o incoming edges
    label and remove v;
    TopologicalSort;
Algorithm (continued)

Simulate deletion of sources using indegree counters

\[
\textbf{TopSort}(\text{Vertex } v); \\
\text{label } v; \\
\textbf{foreach} \text{ edge}(v,w) \\
\quad \text{indeg}(w) = \text{indeg}(w) - 1; \\
\quad \textbf{if} \ \text{indeg}(w) = 0 \\
\quad \quad \textbf{TopSort}(w); 
\]

1. Compute indeg\((v)\) for all vertices
2. Foreach vertex \(v\) do
   \[
   \text{if } v \text{ not labeled and indeg}(v) = 0 \\
   \text{then TopSort}(v) 
   \]